A Bayesian Latent Variable Mixture Model for Filtering Firms' Profit Rates, Income and Capital Stock

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Abstract

By using Bayesian Markov chain Monte Carlo methods we select the proper subset of competitive firms and find striking evidence for Laplace shaped firm profit rate distributions. Our approach enables us to extract more information from data than previous research. We filter US firm-level data into signal and noise distributions by Gibbs-sampling from a latent variable mixture distribution, extracting a sharply peaked, negatively skewed Laplace-type profit rate distribution. A Bayesian change point analysis yields the subset of large firms with symmetric and stationary Laplace distributed profit rates, suggesting a statistical equilibrium of the profit rate at the economy-wide and sectoral levels.

Keywords: Gibbs sampler, Statistical equilibrium, Laplace distribution, Equalization of profit rates

1 Introduction

The formation of a general rate of profit, around which profit rates gravitate, was stressed by classical political economists beginning with Adam Smith [8], who theorized that through competition and capital mobility a tendency of the equalization of profit rates across all competitive industries would emerge. The shape of actual firm level profit rate distributions is little known, however. Apart from [1] and [2], who find Laplace distributed profit rates in two restrictively trimmed datasets of only very large or long-lived firms, we are unaware of empirical estimates at the firm level. We use Bayesian computational methods to select the subset of firms for analysis of firm profitability under competition. Gibbs sampling from a mixture model that assigns a latent variable to every profit rate observation, we sort firms into competitive "signal" and uncompetitive "noise" subsets, without having to resort to essentially arbitrary dropping of data conditional on size or age of firms.

2 Results

Application to the COMPUSTAT database of annual observations of all U.S stock market listed firms 1962-2012, shows the signal profit rate cross sectional distribution is asymmetric Laplace; the NAICS sectoral distributions are also asymmetric Laplace, with very similar modes. A Bayesian change point analysis [3] of the profit rates' first moment on the partition of capital stock trims the data further to a subset of large firms whose first moment is stable. Large firms are stationary symmetric Laplace distributed for the entire time period (Figure 1), which provides strong evidence for a statistical equilibrium in the profit rate distribution among large firms in this data set. Sectoral distributions display very similar modes (Figure 2). Recovery of the empirical transition probabilities confirms that firms to switch from above to below the average profit rate and back confirms profit rate "gravitation".

The previous results are self-contained, but we also hope to present currently ongoing, additional research: we are investigating making the Gibbs sampler algorithm faster by replacing the numerical Laplace mean likelihood integral by a closed form. [6] show this for a Laplace distribution with mean zero by using a mixture representation of exponential family distributions, we are adding a location parameter to be estimated. Furthermore, we are decomposing the profit rate observations into income and capital stock of which the profit rate is the ratio. The resulting two-dimensional distribution gives a richer scaffolding on which to build an economic, statistical model of firm behaviour.

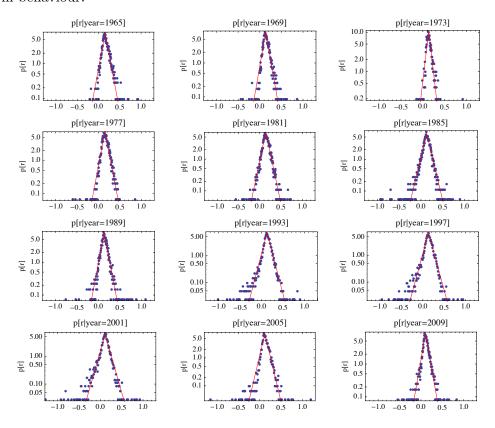


Figure 1: Profit rate distribution of large firms conditional on year of observation, lines are ML fits in the class of Laplace distributions, y-scale logarithmic

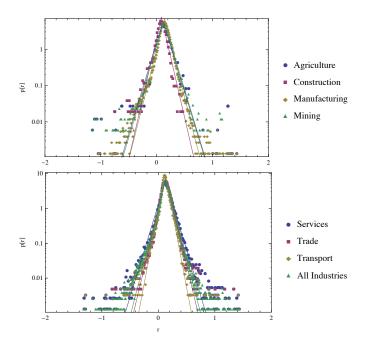


Figure 2: Sectoral Equalization of Profit Rates, data pooled in time domain, lines are ML fits in the class of Laplace distributions, y-scale logarithmic

3 Contributions

Firstly, we confirm previous findings by [1] and [2] of Laplace distributed profit rates for large firms for a much larger economy-wide dataset; on the sectoral level we extend the growth rates results of [5] to profit rates. The striking evidence for a stationary large firm economy-wide distribution and intersectoral equalization of modes calls for a theoretical explanation of the dynamics of individual firms' profit rates that lead to a stable, non-normal distribution, which we are currently investigating with the decomposed dataset. The skew in smaller firms, that disappears for larger firms, raises important questions about entry and exit dynamics.

Secondly, our model enables us to extract more information than previous firm distribution research that used theoretically unjustified, exogenously chosen, small subsets of only large or long-lived firms by [2] for profit rates and e.g. by [9, 4, 7] for growth rates. For our dataset, the competitive firm subset comprises 97% of all observations regardless of age or size. Even after the subsequent change point analysis we discard only 45% of our original data compared to [2], who discard 79% of their data. The better information conservation should make this approach attractive for other applications.

References

- [1] Alfarano, S., Milaković, M. (2008). "Does classical competition explain the statistical features of firm growth?" *Economics Letters*, **101**(3), 272–74.
- [2] Alfarano, S., Milaković, M., Irle, A., Kauschke, J. (2012). "A statistical equilibrium model of competitive firms." *Journal of Economic Dynamics and Control*, 36(1), 136–49.

- [3] Barry, D., Hartigan, J. (1993). "A Bayesian Analysis for Change Point Problems." JASA, 88(421), 309–19.
- [4] Bottazzi, G., Dosi, G., Lippi, M., Pammolli, F., Riccaboni, M. (2001). "Innovation and corporate growth in the evolution of the drug industry." *International Journal of Industrial Organization*, **19**(7),1161–87.
- [5] Bottazzi, G., Secchi, A. (2003). "Common properties and sectoral specificities in the dynamics of u.s. manufacturing companies." *Review of Industrial Organization*, **23**(3-4), 217–32.
- [6] Kozumi, H., Kobayashi, G. (2011). "Gibbs sampling methods for Bayesian quantile regression." Journal of Statistical Computation and Simulation, textbf81(11), 1565– 78.
- [7] Malevergne, Y., Saichev, A., Sornette, D. (2013). "Zipfs law and maximum sustainable growth." *Journal of Economic Dynamics and Control*, **37**(6), 1195–1212.
- [8] Smith, A. (1982)[1776]. The Wealth of Nations. Penguin, London and New York.
- [9] Stanley, M., Amaral, L., Buldyrev, S., Havlin, S., Leschhorn, H., Maass, P., Salinger, M., Stanley, H. (1996). "Scaling behavior in the growth of companies." *Nature*, 379(6568), 804806.