

# Exploring the Uncertainty of a Dendroclimatology Model Using Bayesian Sensitivity Analysis

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## Abstract

A nonlinear forward model, VSLite, simulates tree ring widths using climate data as inputs. There is uncertainty in the inputs which influence the model outputs. Initial comparison of the distributions of inter-annual ring widths from real trees with those simulated by VSLite, suggests that the latter exhibits considerably more variability than the former. In order to explore this, we conducted a Bayesian sensitivity analysis (BSA) of VSLite using a Gaussian process emulator. Such an analysis allows us to understand and quantify the uncertainty of the model's outputs due to changes in its inputs. In this talk we will report on our experiments and explore why and how the observed differences occur.

**Keywords:** VSLite model; model uncertainty; probabilistic sensitivity analysis; gaussian process.

## 1 Background

The VSLite model [3] is a nonlinear forward model which relates climate to tree-ring widths. This simple model allows simulation of tree-ring width sequences at any geographic location around the world for which monthly records of temperature and precipitation exist. In addition to the site latitude and climate records, the model requires specification of a further 13 parameters relating to the ways in which climatic factors limit tree-ring growth (e.g. via changes in soil moisture). The model is freely available from (<http://www.ncdc.noaa.gov/paleo/softlib.html>). A Bayesian scheme is provided by [4] for estimating the four main growth response parameters of the model at a single geographic location given monthly temperature, accumulated precipitation, site latitude, and tree-ring width data.

Raw ring-width data for sites of interest can be obtained from the International Tree-Ring Data Base (ITRDB, <http://www.ncdc.noaa.gov/paleo/treering.html>), and these data are detrended (using well established methods) to remove age-trends, thus producing sequences of tree-ring indices over decades or centuries. Historic precipitation and temperature records for locations of interest can also be obtained from the British Atmospheric Data Centre (<http://www.badc.nerc.ac.uk/browse/badc>).

## 2 Exploration of variability and uncertainty

Exploration of the residuals between sequences of real ring-width indices and those simulated by VSLite (at the same geographic location) suggests that VSLite simulates tree-ring sequences with greater variability than that seen between real trees. Given this, interest focuses on the various sources of variability within the model and why these are so large.

In this presentation we will describe our work to perform a Bayesian sensitivity analysis (BSA) on VSLite (as suggested by [1]). Our goal is to understand and quantify how changes in the output of the model can be attributed to changes in the input parameters.

Under the BSA, each uncertain input parameter of the model is represented as a probability distribution, and then a Gaussian process is fitted using a set of training data points which are obtained from running VSLite at carefully chosen input points. The BSA approach consists of two main stages. The first is to build a statistical emulator for VSLite given a set of training runs of the model. The second is to use the created emulator to calculate sensitivity indices for inputs of interest. Unlike widely used methods of SA, described in [2], which are computationally expensive and depend on Monte Carlo sample-based methods to estimate sensitivity indices, the BSA approach is simpler and no additional runs of the original model are needed to compute sensitivity indices.

## References

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