Growth Determinants and Bayesian Non-linear Estimation of Threshold Inflation

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Abstract

Threshold inflation is the rate above which the increase in prices is detrimental to economic growth. A Bayesian non-linear estimator of threshold inflation is proposed. Bayesian Model Averaging was used to account for the uncertainty about the determinants of economic growth. The Bayesian threshold has a direct application for central banks with multiple mandates.

Keywords: Bayesian threshold estimation; BMA; growth determinants; new trends of policies in central banking

1 Introduction

Due to the recent international financial and economic crises, now it is thought that monetary policy cannot ignore the real sector of the economy. There exist a case in favor of raising the inflation target to avoid the zero lower bound of interest rates and thus have more space to implement an expansionary policy during an economic contraction; see *inter alia* Williams (2009), Blanchard et al. (2010), Walsh (2011) or Issing (2011). This research argues that the limit to raise the inflation target is *threshold inflation*, defined as the inflation rate above which inflation has a detrimental effect on economic growth. A Bayesian method for estimating this threshold is proposed. An application is shown in Section 3.

2 A Non-Linear BMA Estimation of Threshold Inflation

Let n be the sample size and $\mathbf{y} = (y_1, \ldots, y_n)$ be data on economic growth. The k explanatory variables of growth are contained in a $n \times (k+1)$ matrix $\mathbf{X}$. In the ordinary Gaussian lineal regression model,

$$\mathbf{y}|\beta, \sigma^2, \mathbf{X} \sim \mathcal{N}_n\left(\mathbf{X}\beta, \sigma^2\mathbf{I}_n\right), \quad \mathbb{E}\left[y_i|\beta, \mathbf{X}\right] = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik}, \quad \mathbb{V}\left(y_i|\sigma^2, \mathbf{X}\right) = \sigma^2.$$

Under quadratic loss, the Bayesian optimal point estimators of β y σ^2 are,

$$\begin{aligned}\mathbb{E}\left[\beta|\mathbf{y}, \mathbf{X}\right] &= \mathbb{E}\left[\mathbb{E}(\beta|\sigma^2, \mathbf{y}, \mathbf{X})|\mathbf{y}, \mathbf{X}\right] \\ &= \left(M + \mathbf{X}^T\mathbf{X}\right)^{-1}\left\{(\mathbf{X}^T\mathbf{X})\hat{\beta} + M\tilde{\beta}\right\},\end{aligned}$$

and (for $n \geq 2$),

$$\mathbb{E}\left[\sigma^2|\mathbf{y},\mathbf{X}\right] = \frac{2b + s^2 + \left(\tilde{\beta} - \hat{\beta}\right)^T \left(M^{-1} + \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\right)^{-1} \left(\tilde{\beta} - \hat{\beta}\right)}{n + 2a - 2}.$$

Bayesian estimators with Zellner's g-priors. Zellner (1986) suggested the use of an informative g-prior for $a, b, \tilde{\beta}, M$, based on a conditional gaussian prior for β,

$$\beta|\sigma^2, \mathbf{X} \sim \mathcal{N}_{k+1}\left(\tilde{\beta}, g\sigma^2\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\right),$$

and an improper Jeffrey's prior $\mathbb{P}\left(\sigma^2|\mathbf{X}\right) \propto \sigma^{-2}$ for σ^2. With Zellner's g-prior, the point estimators are,

$$\mathbb{E}\left[\beta|\mathbf{y},\mathbf{X}\right] = \frac{1}{g+1}\left(\tilde{\beta} + g\hat{\beta}\right),$$

$$\mathbb{E}\left[\sigma^2|\mathbf{y},\mathbf{X}\right] = \frac{s^2 + \left(\tilde{\beta} - \hat{\beta}\right)^T \mathbf{X}^T\mathbf{X}\left(\tilde{\beta} - \hat{\beta}\right)/(g+1)}{n-2},$$

$$\mathbb{V}\left[\beta|\mathbf{y},\mathbf{X}\right] = \frac{g}{g+1}\frac{s^2 + \left(\tilde{\beta} - \hat{\beta}\right)^T \mathbf{X}^T\mathbf{X}\left(\tilde{\beta} - \hat{\beta}\right)/(g+1)}{n}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}.$$

Non-linear models. Let τ be the value above which inflation has a changing effect on economic growth. Letting the effect of inflation be continuous with $\pi^s = \ln(\pi) - \ln(\tau)$, the non-linear relation between growth and inflation can be captured with,

$$\mathbf{\Pi}_{(\tau)} = \begin{bmatrix}\ln(\pi) & \mathbf{1}_{(\pi>\tau)}\pi^s\end{bmatrix},$$

where $\mathbf{1}_{(\pi>\tau)}$ is a binary vector with values equal to one when inflation is higher than τ and zero in other cases. With a matrix of control variables $\mathbf{C}$,

$$\mathbf{y}|\gamma_{(\tau)}, \phi_{(\tau)}, \sigma^2_{(\tau)}, \mathbf{\Pi}_{(\tau)}, \mathbf{C} \sim \mathcal{N}_n\left(\mathbf{\Pi}_{(\tau)}\psi_{(\tau)} + \mathbf{C}\phi_{(\tau)}, \sigma^2_{(\tau)}\mathbf{I}_n\right),$$

then,

$$\mathbb{E}\left(\mathbf{y}|\psi_{(\tau)}, \phi_{(\tau)}, \mathbf{\Pi}_{(\tau)}, \mathbf{C}\right) = \mathbf{\Pi}_{(\tau)}\psi_{(\tau)} + \mathbf{C}\phi_{(\tau)}, \qquad \mathbb{V}\left(y_i|\sigma^2_{(\tau)}, \mathbf{X}_{(\tau)}, \mathbf{C}\right) = \sigma^2_{(\tau)}.$$

Non-linear Bayesian Estimators. Let $\mathbf{X}_{(\tau)} = [\mathbf{\Pi}_{(\tau)} \quad \mathbf{C}]$ and $\beta_{(\tau)} = [\psi \quad \phi_{(\tau)}]$. The threshold non-linear model can be expressed as,

$$\mathbf{y}|\beta_{(\tau)}, \sigma^2_{(\tau)}, \mathbf{X}_{(\tau)} \sim \mathcal{N}_n\left(\mathbf{X}_{(\tau)}\beta_{(\tau)}, \sigma^2_{(\tau)}\mathbf{I}_n\right),$$

with,

$$\mathbb{E}\left(\mathbf{y}|\beta_{(\tau)}, \mathbf{X}_{(\tau)}\right) = \mathbf{X}_{(\tau)}\beta_{(\tau)}, \qquad \mathbb{V}\left(y_i|\sigma^2_{(\tau)}, \mathbf{X}_{(\tau)}\right) = \sigma^2_{(\tau)}.$$

For a known value of τ, the Bayesian point estimators are,

$$\mathbb{E}\left[\beta_{(\tau)}|\mathbf{y}, \mathbf{X}_{(\tau)}\right] = \frac{1}{g+1}\left(\tilde{\beta} + g\hat{\beta}_{(\tau)}\right),$$

$$\mathbb{V}\left[\beta_{(\tau)}|\mathbf{y}, \mathbf{X}_{(\tau)}\right] = \frac{g}{g+1}\frac{s^2 + \left(\tilde{\beta} - \hat{\beta}_{(\tau)}\right)^T \mathbf{X}^T_{(\tau)}\mathbf{X}_{(\tau)}\left(\tilde{\beta} - \hat{\beta}_{(\tau)}\right)/(g+1)}{n}\left(\mathbf{X}^T_{(\tau)}\mathbf{X}_{(\tau)}\right)^{-1},$$

and the marginal likelihood of the threshold non-linear model is,

$$\mathscr{L}\left(\mathbf{y}|\mathbf{X}_{(\tau)}\right) = (g+1)^{(k+1)/2}\pi^{-n/2}\Psi(n/2)\left[\mathbf{y}^T\mathbf{y} - \frac{g}{g+1}\mathbf{y}^T\mathbf{X}_{(\tau)}\left(\mathbf{X}_{(\tau)}^T\mathbf{X}_{(\tau)}\right)^{-1}\mathbf{X}_{(\tau)}^T\mathbf{y} - \frac{1}{g+1}\tilde{\beta}_{(\tau)}^T\mathbf{X}_{(\tau)}^T\tilde{\beta}_{(\tau)}\right]^{-n/2}.$$

Bayesian estimator of threshold inflation under uncertainty about growth determinants. A BMA non-linear point estimator of the threshold τ is obtained by adding the estimates of each model τ_j, weighted by the posterior probability of each model being true $\mathbb{P}\left(\mathcal{M}_j|\mathcal{D}\right)$,

$$\mathbb{E}\left(\tau|\mathcal{D}\right) \propto \sum\nolimits_{\mathcal{A}_\tau}\mathbb{P}\left(\mathcal{M}_j|\mathcal{D}\right)\tau_j,$$

for a subset $\mathcal{A}_\tau \subset \mathcal{M}$. See Lubrano (1998), Koop and Potter (2003), Koop et al. (2007) or Oleg (2009).

3 Application

The annual inflation rate and the economic growth of Bolivia for $n = 41$ years was used as a case study. Besides inflation, the set of $k = 9$ control variables includes data on fertility rates, population growth, the share of extractive activities in output, life expectancy at birth, gross fixed capital formation, terms of trade, primary school enrollment and social conflicts.

A sensitivity analysis was performed using three types of g-priors ($g = 1/k^2$, $g = \sqrt{1/n}$, $g = 1$) and two types of prior probabilities of a model being true $\mathbb{P}\left(\mathcal{M}_j\right)$, uniform or binomial.

Table 1 shows the Bayesian estimates of threshold inflation. In the case of a uniform prior, the threshold is about 11%, while with a binomial prior the threshold is around 9%.

Table 1: Threshold Inflation τ^a

	Prior $\mathbb{P}\left(\mathcal{M}_j\right)^b$	
g-prior	Uniform	Binomial
$1/k^2$	11.49	8.92
$\sqrt{1/n}$	11.36	8.90
1	10.84	8.85

[a] In percentage
[b] $\mathbb{P}\left(\mathcal{M}_j\right)$: Prior probability of each model being true

The scatter-plot of inflation and output (Figure 1) shows the strong inverse relationship between output and the change in prices when inflation is above the threshold.

4 Discussion

A Bayesian approach for the estimation of threshold inflation was proposed. The results were robust to the choice of different priors and are consistent both with theoretical

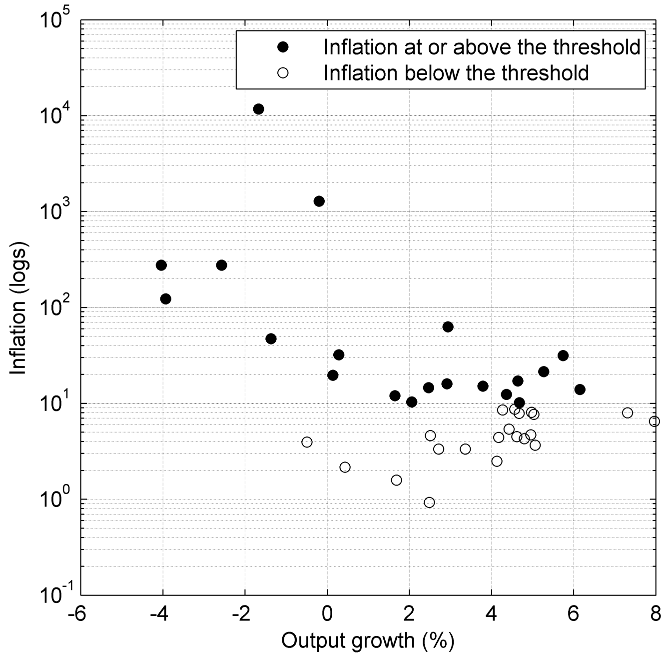

Figure 1: Scatter-plot of output growth and inflation.

vision of Gillman and Kejak (2005) and the empirical evidence of *inter alia* Barro (1995), Bruno and Easterly (1998), De Gregorio (1992), Dabus and Viego (2003) or Bittencourt (2012).

The Bayesian estimation of the threshold allows to include economic growth into the mandates of a monetary authority: with a measure of *threshold inflation* (π^u), the period loss function of monetary policy is a conditional function,

$$L_t = \begin{cases} \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda \tilde{y}_t^2\right] & \text{if} \quad 0 \leq \pi_t \leq \pi^u, \\ \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda \tilde{y}_t^2 + \omega \Delta y_t^2\right] & \text{if} \quad \pi_t > \pi^u, \end{cases}$$

with a inter-temporal loss function,

$$\mathscr{U}_t = \begin{cases} \mathbb{E}_t\left(\sum_{j=0}^{\infty} \delta^j \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda \tilde{y}_t^2\right]\right) & \text{if} \quad 0 \leq \pi_{t+j} \leq \pi^u, \\ \mathbb{E}_t\left(\sum_{j=0}^{\infty} \delta^j \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda \tilde{y}_t^2 + \omega \Delta y_t^2\right]\right) & \text{if} \quad \pi_{t+j} > \pi^u. \end{cases}$$

where $\lambda \geq 0$ is the weight on output-gap stabilization $\tilde{y}_t$, $\delta \in (0,1)$ is a discount factor and $\mathbb{E}_t$ is an expectation operator conditional on information available in year t (Svensson, 1999). In this equation, economic growth is included into the mandates of a Central Bank, thus if observed inflation (π_t) or anticipated inflation (π_{t+j}) is higher than the threshold (π^u), then the monetary authority assigns a relative importance ($\omega > 0$) to economic growth Δy_t^2, besides inflation and output-gap stability. Below the threshold, if $\lambda = 0$, the monetary authority only cares about the long-run target π^*, while if $\lambda > 0$, output-gap stabilization is also a concern of a monetary authority, as in models of flexible inflation targeting.

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